University of Illinois Urbana-Champaign
PHYS 398: Design Like a Physicist

Quantitative Analysis on the Tonal Quality of Various Pianos

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ABSTRACT

The frequency of all notes on several grand pianos are recorded and analyzed. The dissonant effect is used to determine the influence of octave correspondence, overtone amplitude, and self-dissonance on the tonal quality of a piano. In this paper, tonal quality is defined as how in-tune a piano is.

INTRODUCTION

Piano tuning did not arise as a mainstream profession until the early 19th century, almost an entire century after the invention of the piano itself. Traditionally, technicians would tune pianos using "Pythagorean" and "just intonation" methods, which employ frequency ratios in order to create harmony. These methods were often modified to increase the consonance of certain intervals, but they came with the disadvantage of increased dissonance in other intervals.[1] Each tuning method relied on a different set of frequencies, and as a result, there was no consistent interval size for a piano.

Eventually, 12-tone equal temperament was adopted in Western classical music for its convenience with modern piano design and minimized dissonance. In 1938, however, O.L. Railsback observed a divergence in frequencies of recently tuned pianos from the standard equal temperament. Upon measuring the frequencies of each of the 88 keys, Railsback determined that technicians tend to "stretch" octaves when tuning a piano; that is, lower octaves are tuned slightly flat and upper octaves are tuned slightly sharp.[2]

Over time, pianos fall out of tune as a result of changes in barometric pressure, temperature, and string tension. Traditionally, a piano is considered "out of tune" if the frequencies of its keys deviate noticeably from where they were set by a technician. The perceived changes in tonal quality may depend on more than a simple frequency shift.

Scientific Basis

The cochlear duct is a series of fluid-filled chambers located in the inner ear responsible for auditory perception (Figure 0). Within the cochlear duct is the Organ of Corti, which acts as nature's microphone, transforming pressure waves into electrical nerve signals, i.e. converting sound to messages processable by the brain.[3] The frequency range of human hearing sits slightly above average across the animal kingdom, ranging from ~20–20,000 Hz (comparatively, elephants can hear frequencies ranging from ~17–10,500 Hz), the sensitivity of which is determined by discrete cilia movement.[4]

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[1] Berg, R.E.; Stork, D.G. (2005). *The Physics of Sound* (3rd ed.), Pearson Education Inc.

[2] Giordano, N. (2015, October 23). *Explaining the Railsback stretch in terms of the inharmonicity of piano ....* The Journal of the Acoustical Society of America. Retrieved April 4, 2019, from https://asa.scitation.org/doi/10.1121/1.4931439

[3] (2016, August 10). Ear, middle ear, cochlea, | Cochlea - Cochlea.org. Retrieved April 4, 2019, from http://www.cochlea.org/en/hearing/ear

[4] RR Fay. Hearing in vertebrates: A psychophysics databook. - APA PsycNET. Retrieved April 4, 2019, from http://psycnet.apa.org/record/1988-98268-000

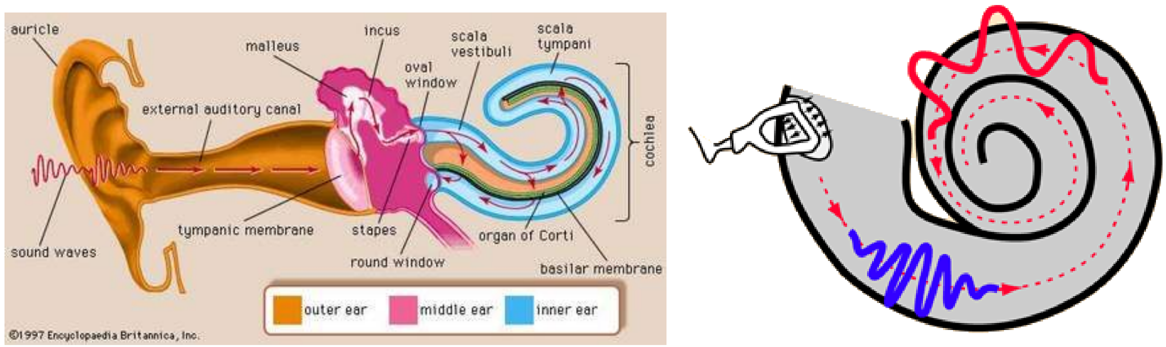

Figure 0: Inner ear diagram (left) showing cochlear duct anatomy and Organ of Corti location. Cochlear duct anatomy (right) showing sounds of different frequencies traveling different distances within the Cochlear duct.

Place Theory suggests that sounds of different frequencies travel different distances within the Cochlear duct before maximally exciting the cilia. High frequencies find maximal excitation near the entrance of the duct and low frequencies travel deeper into the spiral (Figure 0)[5]. This behavior gives rise to critical bands characterized by a central frequency f_0 and a bandwidth. For frequency and loudness perception, human hearing is logarithmic rather than linear, and as a result, bandwidths are not consistent across frequencies. For frequencies <1,000 Hz, bands span 100 Hz, and for frequencies >1,000 Hz, bands span ~15% of the central frequency (Figure 1).

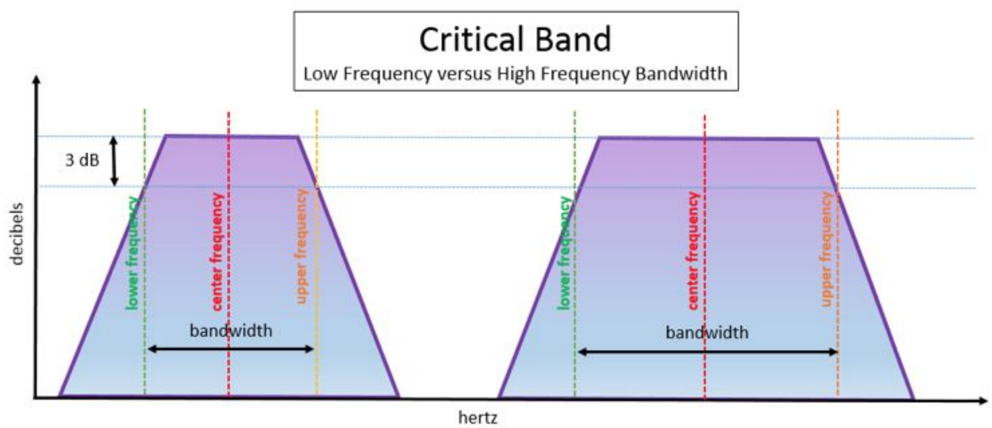

Figure 1: Diagram of critical bands defined by a center frequency and bandwidth. For low frequencies, bands span 100 Hz. For high frequencies, bands span ~15% of the center frequency.

When two notes have frequencies within the same critical band, they are perceived as dissonant, with maximum dissonance at a separation of ~30% of the critical bandwidth. If the notes are brought sufficiently close together (<15 Hz), dissonance disappears but beating between the two notes arises until eventually the frequencies become identical.[6]

[5] (n.d.). The Place Theory of Pitch Perception - HyperPhysics Concepts. Retrieved April 19, 2019, from http://hyperphysics.phy-astr.gsu.edu/hbase/Sound/place.html

[6] (1999, December 1). Consonance and Dissonance. Retrieved April 4, 2019, from http://hep.physics.indiana.edu/~rickv/consonance_and_dissonance.html

In an equal temperament scale, the frequencies of two successive notes are separated by a multiplicative factor of $\sqrt[12]{2} \approx 1.059463$. More generally

$$f_n = f_a(\sqrt[12]{2})^{(n-a)} \tag{1.0}$$

where f_n is the frequency (pitch) of the nth note, f_a is the frequency of a reference note, and n and a are integers referring to the number of keys from the left end of a piano (e.g. A_4 is the 49th key from the left end of a piano and is assigned the integer 49).

A pure tone is characterized by a sine wave with a single frequency. Pianos do not produce pure tones; rather, they produce complex tones consisting of multiple frequencies above a fundamental pitch, known as harmonics. In an ideal system, the complex tone of a certain fundamental frequency f_0 will include harmonics whose frequencies are integer multiples of f_0, i.e.

$$f_n = nf_0 \tag{1.1}$$

where f_n is the nth harmonic and n is an integer. Therefore, in analyzing the dissonance of two notes, it is necessary to consider both the fundamental frequencies of the notes as well as their harmonic frequencies. The harmonics of a note not only help determine dissonance but factor into the tonal quality of the note as well, with a greater presence of harmonics creating a "brighter," "darker," or "richer" tone.

In an ideal system, harmonics abide by equation 1.0, but pianos cannot be approximated as ideal systems because they exhibit "inharmonicity." Inharmonicity can be understood as a deviation of frequencies from their expected values, caused by the acoustical impedance of the piano, i.e. the rigidity of the piano structure does not propagate sound waves efficiently and leads to pitch shifting. For sake of clarification, frequency-shifted harmonics resulting from inharmonicity are referred to as "partials" or "overtones". With inharmonicity, the frequency of the nth partial is

$$f_n = nf_0\sqrt{\tfrac{1+Bn^2}{1+B}} \tag{1.2}$$

where f_0 is the fundamental frequency (also referred to as the "first partial" or "first overtone") and B is a constant dependent on string diameter and length.[7] While it might seem straightforward to tune each note to its frequency calculated from equal temperament, piano tuners must actually stretch notes from these frequencies to account for inharmonicity, leading to the Railsback Curve (Figure 2).[8]

[7] Young, R.W. (1952). *Inharmonicity of Plain Wire Piano Strings*. Journal of the Acoustical Society of America. Retrieved April 4, 2019, from https://asa.scitation.org/doi/10.1121/1.1906888
[8] Railsback, O.L. (1938). *A Study of the Tuning of Pianos*. Journal of the Acoustical Society of America. Retrieved April 4, 2019, from https://asa.scitation.org/doi/10.1121/1.1902080

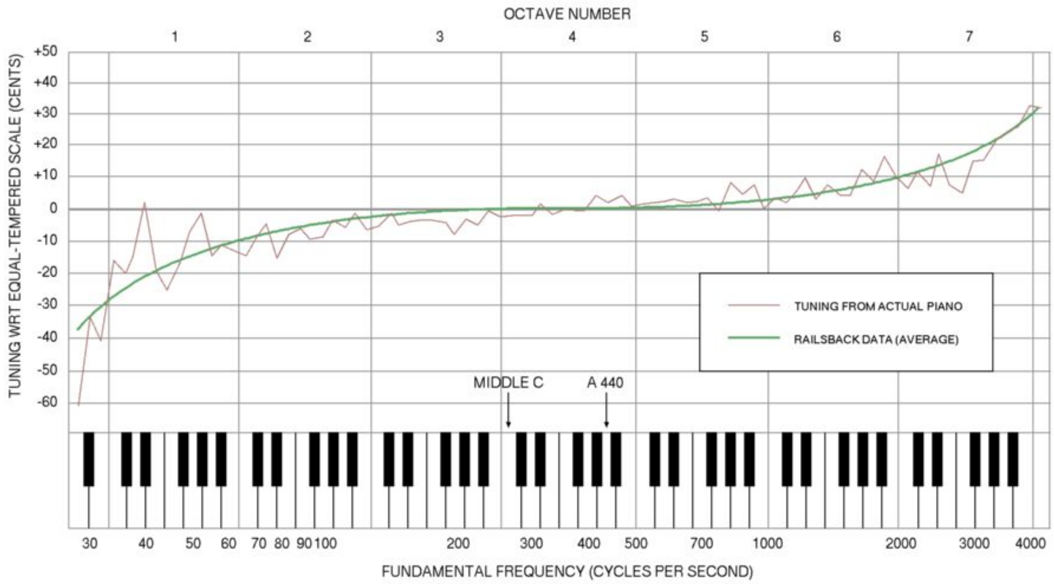

Figure 2: The Railsback Stretch. Deviation from equal temperament is measured in cents (1/100th of the frequency difference between two semitones) on the vertical axis and frequency is measured on the horizontal axis.

The goal of this paper is to determine quantitatively what characterizes a piano that is "out of tune." It is well known that a change in frequency of the strings will cause a piano to sound out of tune, and the analysis will investigate and expand upon this. The frequency spectra of several tuned and untuned pianos were investigated.

METHODS

Following a breadboard prototype (Figure 3), a printed circuit board (PCB) was used to record pianos. An Arduino microcontroller and various sensors, including an electret microphone, liquid crystal display (LCD), keypad, current sensor, mono amplifier, real-time clock, integrated environmental sensor, and a secure digital (SD) breakout (Figure 4) are attached to the PCB.

As sound propagates through the air in the form of waves, a thin, flexible diaphragm moves to create varying current in standard microphones. The electret microphone works on the principle of varying capacitance. Parallel plate capacitors are fixed near a movable plate. When vibrations are detected, a voltage change is induced by the moving diaphragm.

Electret microphones can connect to a Junction Field Effect Transistor to increase the generated current. There are three types of electret microphones depending on the film used for the diaphragm and the location of the moveable plate. These include back electret, front electret, and foil-type. The electret microphone used in this experiment was a foil-type with an audible range of 20-20,000 Hz.

The Arduino records audio through the electret microphone and writes the data as a binary file to the SD card. The audio is recorded at a sampling rate of 32,000 Hz using the Arduino's 10-bit Analog to Digital Converter (ADC). This converter maps input voltages between 0-2.56 Volts to integer values between 0 and 1023.[9] Test recordings were taken to measure the background noise of the electronics. The microphone gain was adjusted to minimize this noise to a value of 1.5 root mean square (rms) counts.

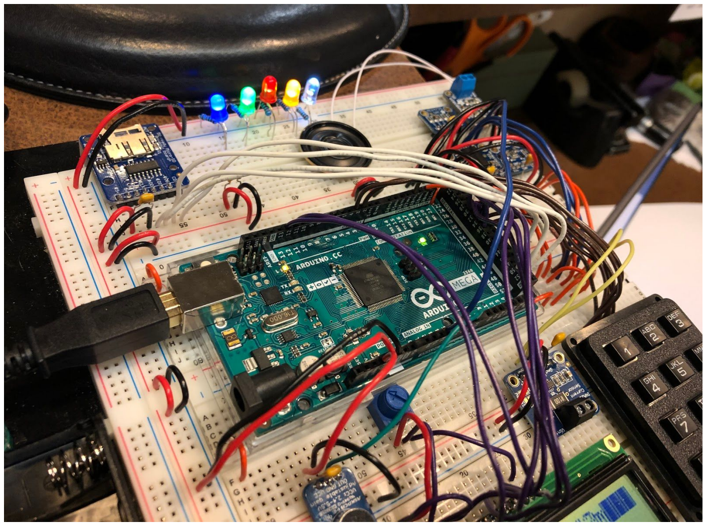

Figure 3: The original Arduino breadboard prototype included the sensors needed for running tests.

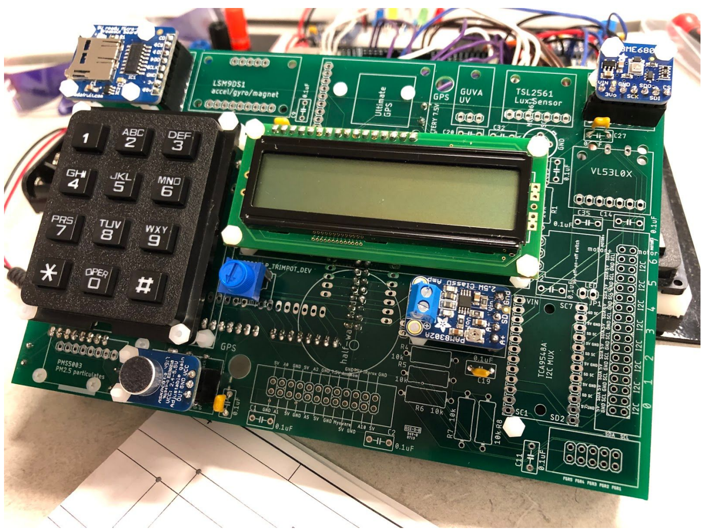

Figure 4: The final Arduino on a printed circuit board was used for the majority of tests. The PCB included only the necessary sensors to run the experiments.

The pianos recorded were the Krannert Great Hall Steinway Concert Grand, Tryon Festival Theatre Steinway Grand, Krannert Orchestra Practice Room Steinway Grand, Krannert Choral Practice Room Steinway Grand, Krannert Practice Room A Mason & Hamlin Grand, Ikenberry Hall Yamaha Upright, Palos Park Yamaha Concert Grand, and Barrington Yamaha Baby Grand.

Originally, all 88 keys were recorded, followed by middle C (C4) played with each pedal depressed one at a time. Through initial analysis, it became clear that subsequent octaves (a set of

[9] "AnalogRead()." *Arduino Reference*, www.arduino.cc/reference/en/language/functions/analog-io/analogread/.

twelve notes starting at C and ending at B) exhibit similar trends; therefore, it was only necessary to record octaves C2, C4, and C6, represented by the orange, green, and indigo regions in Figure 5. Middle C was still recorded with each pedal depressed. In those octaves, every key was recorded, but this analysis focuses only on the white keys. Between the recording of each note, adequate time was given for the sound to dissipate.

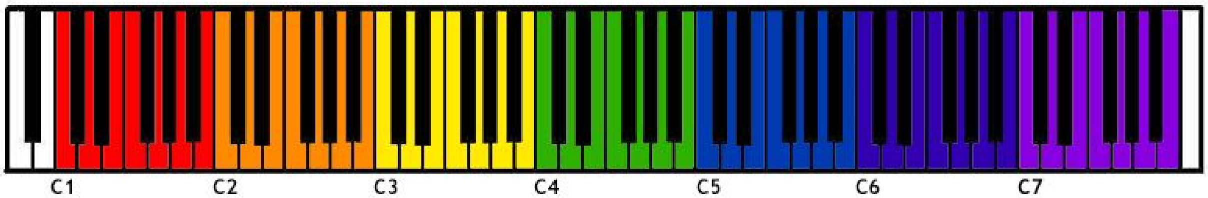

Figure 5: Different octaves of a piano are represented by different colors.

The code used to analyze the recordings was written in Python. First, the code converts the binary files to wav files with the appropriate length buffers to ensure the whole file is converted. The wav file is then plotted on a graph as amplitude versus time to find the duration of each note. Then, every note in the octave is extracted from the wav file and analyzed with a Fast Fourier Transform (FFT). One part of the code calculates a forward Discrete Fourier Transform and returns the coefficients of acoustic power. Another part of the code computes the frequencies corresponding to these coefficients.[10] Using these results, a graph is created to display the frequency spectrum of a note (Figure 6). The same notes from different pianos are overlaid on a graph to show any deviations.

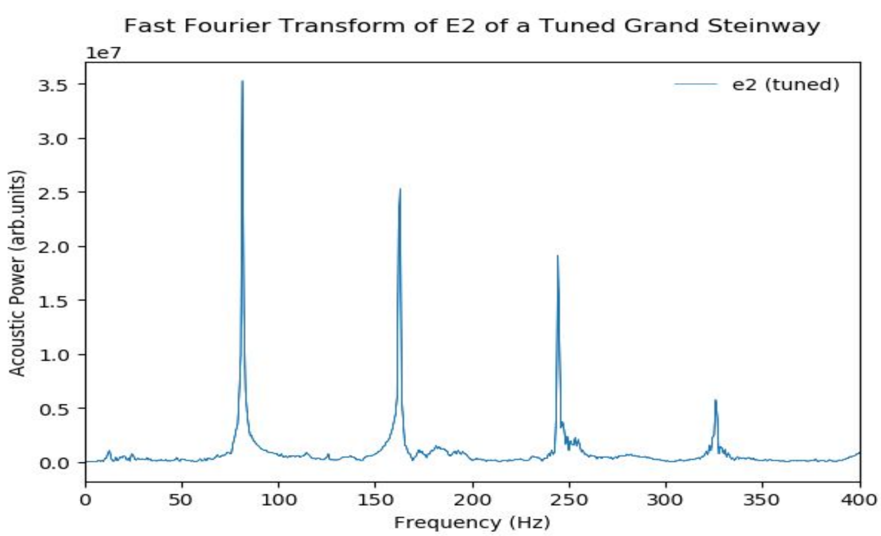

Figure 6: An example graph of a Fast Fourier Transform of E2 on a tuned grand Steinway. The first peak represents the fundamental frequency, and the following peaks represent the harmonics.

Spectrograms plot frequency versus time and use color to indicate the intensity in decibels. An FFT or other type of Fourier Transform are used to display the frequency information. Figure 7

[10] "Fourier Transforms." http://snowball.millersville.edu/~adecaria/ESCI386P/esci386-lesson17-Fourier-Transforms.pdf. Accessed 5 Apr. 2019.

shows a sample spectrogram. Each tone in a chromatic scale is a different column on the plot. The settings of spectrogram gain are 10-80 dB. The notes are well defined between the range of 0-3,000 Hz. Tones above -20 dB are red while tones in blue or magenta are below -20 dB. Sound Pressure Level

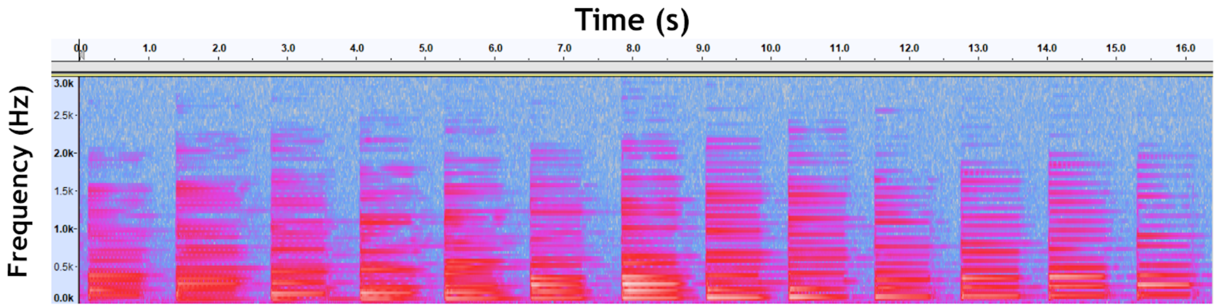

Figure 7: A spectrogram for C2 Chromatic Scale on a tuned Steinway. The amplitudes of the frequencies decrease over time, as indicated by the colors. The intensity is in decibels and is represented by different colors in dB SPL.

Fast Fourier Transform graphs are used to determine the frequency spectrum and strongest harmonic for each note, the results of which are stored in a table for every octave recorded. Various graphs and plots are created as necessary to analyze the data.

RESULTS AND ANALYSIS

Four influences on the tonal quality of a piano were investigated: frequency shifts, octave correspondence, overtone amplitude, and self-dissonance for the frequency spectra produced.

The results of performing an FFT are shown in Figure 8. This graph represents the fundamental frequencies of the C4 octave of the Palos Park Yamaha Concert Grand Piano. The piano was kept in good condition, cleaned regularly, and played rarely.

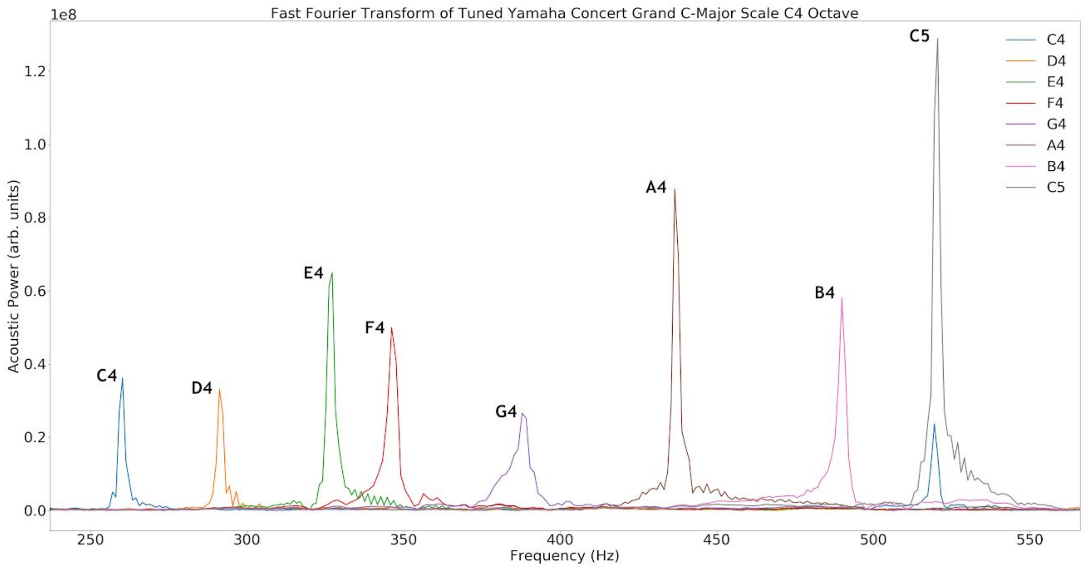

Figure 8: FFT of the C4 octave for a tuned Yamaha. The x-axis is the frequency measured in Hertz and the y-axis is the acoustic power in arbitrary units. Two frequency peaks for the final note in the scale are observed.

Frequency Shifts

As mentioned previously, when the frequency of a note deviates noticeably from its equal temperament frequency, it is perceived as sharp or flat. When a number of piano keys are sharp or flat, the piano as a whole begins to sound out-of-tune.

Figures 9 through 11 compare the fundamental frequencies of D2, D4, and D6 of a tuned and an untuned Grand Steinway. From these graphs, it is clear the fundamental frequencies of D2, D4, and D6 on an untuned piano deviate from their tuned counterparts. Furthermore, as the analysis extends from lower octaves to higher octaves, this deviation grows according to equation 1.1. This trend can be observed for octaves C2, C4, and C6 in Figure 12 where the frequency difference grows from 0.1965 Hz at D2 to 7.31 Hz at D6.

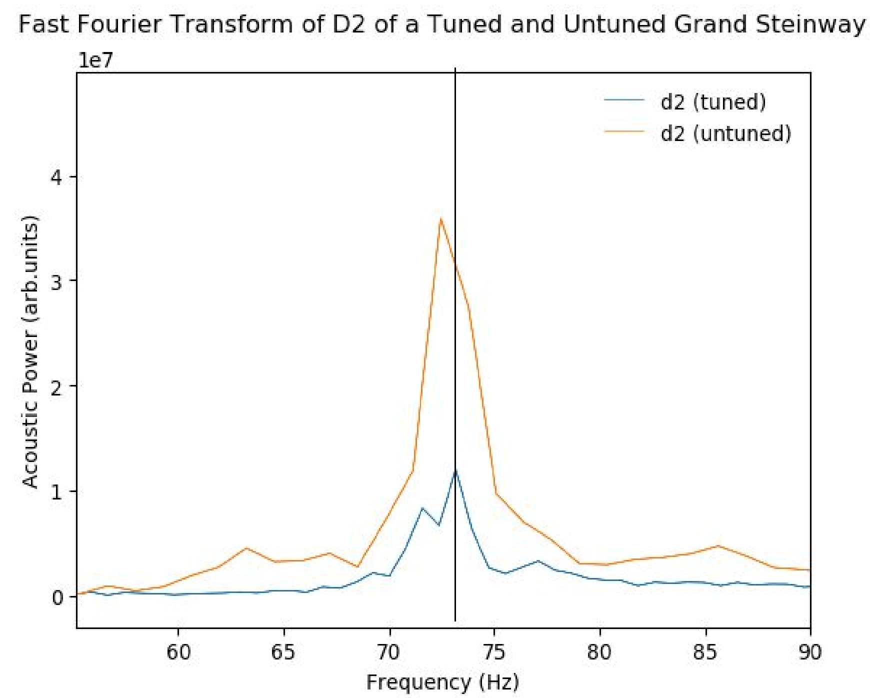

Figure 9: The graph shows a fast fourier transform frequency comparison of D2 on a tuned and untuned Steinway. The x-axis is frequency measured in Hertz and the y-axis is the acoustic power. The frequency difference in peaks is about 0.1965 Hz.

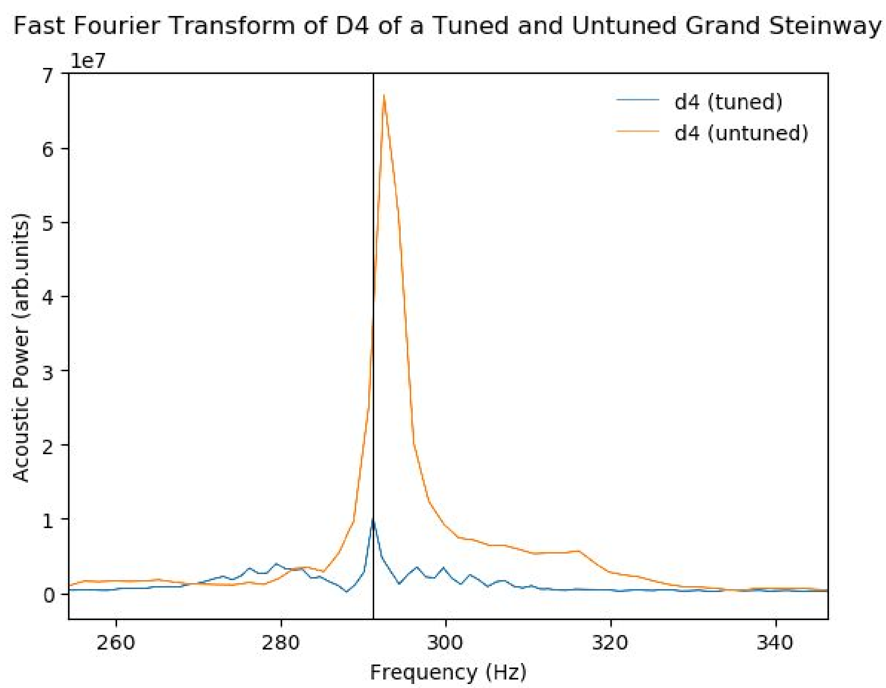

Figure 10: The graph shows a fast FFT frequency comparison of D4 on a tuned and untuned Steinway. The x-axis is frequency measured in Hertz and the y-axis is the acoustic power. The frequency difference in peaks is about 1.27 Hz.

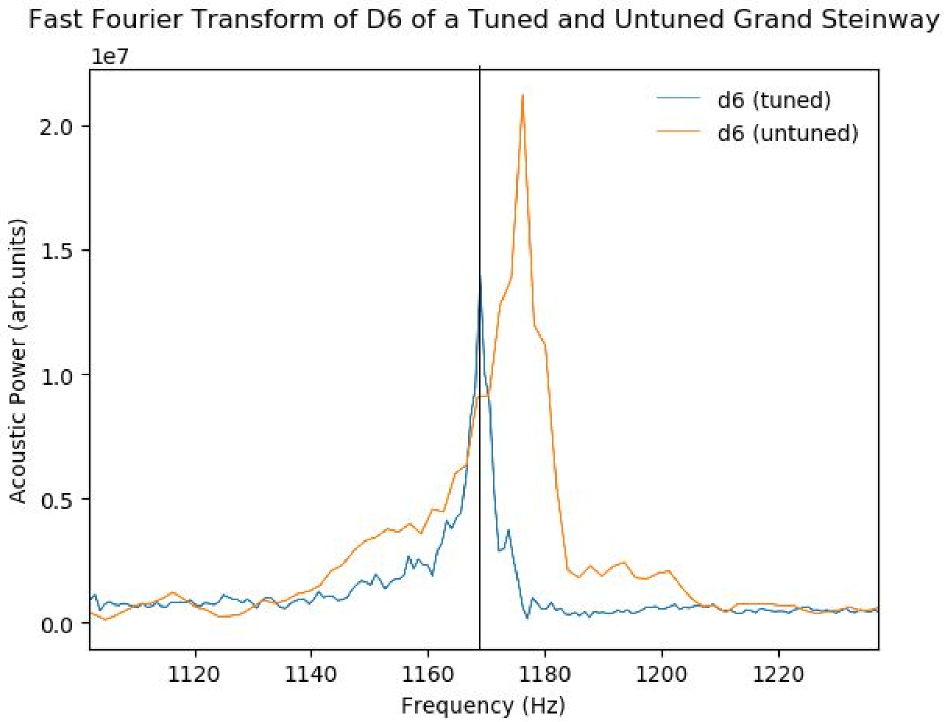

Figure 11: The graph shows a FFT frequency comparison of D6 on a tuned and untuned Steinway. The x-axis is frequency measured in Hertz and the y-axis is the acoustic power. The frequency difference in peaks is about 7.31 Hz.

Note	Grand Steinway (Tuned)	Grand Steinway (Untuned)	
	1st Fundamental (Hz)	1st Fundamental (Hz)	Abs. Diff(Hz)
C2	64.3134	64.7931	0.4797
D2	72.6644	72.4679	0.1965
E2	81.4966	81.9685	0.4719
F2	86.0679	86.5395	0.4716
G2	96.5554	97.1446	0.5892
A2	108.647	110.227	1.58
B2	122.037	123.212	1.175

C4	259.16	260.11	0.95
D4	291.236	292.506	1.27
E4	326.103	329.455	3.352
F4	346.402	349.017	2.615
G4	389.026	391.21	2.184
A4	437.008	439.047	2.039
B4	489.549	491.977	2.428
C5	519.385	518.113	1.272
C6	1042.58	1048.58	6
D6	1168.99	1176.3	7.31
E6	1313.78	1322.94	9.16
F6	1393.56	1400.1	6.54
G6	1564.4	1576.34	11.94
A6	1758.03	1770.13	12.1
B6	1973.52	1990.78	17.26

Figure 12: Quantitative results for the fundamental frequencies and the calculated absolute difference between the tuned and untuned fundamental frequencies of the Grand Steinway pianos in Figure 7-9.

Octave Correspondence

Octave correspondence is the primary method used to tune pianos by aligning the frequencies of the second harmonic of C4 with the first harmonic of C5. Figures 13 through 16 compare the fundamental frequencies and the harmonic frequencies of C5 with the harmonic frequencies of C4 in a tuned and an untuned Grand Steinway. For a tuned piano, the second harmonic frequency of C4 clearly aligns with the fundamental frequency of C5 (Figures 13 and 15). In contrast, an untuned piano exhibits a significant difference in frequencies between the two notes (Figures 14 and 16).

The frequency spectrum of a low note is compared to that of a higher note to characterize the possible results of dissonance from an out of tune piano. The C4 harmonics with the C5 fundamentals and the harmonics of both tuned and untuned piano were compared by overlaying the corresponding frequency spectrum.

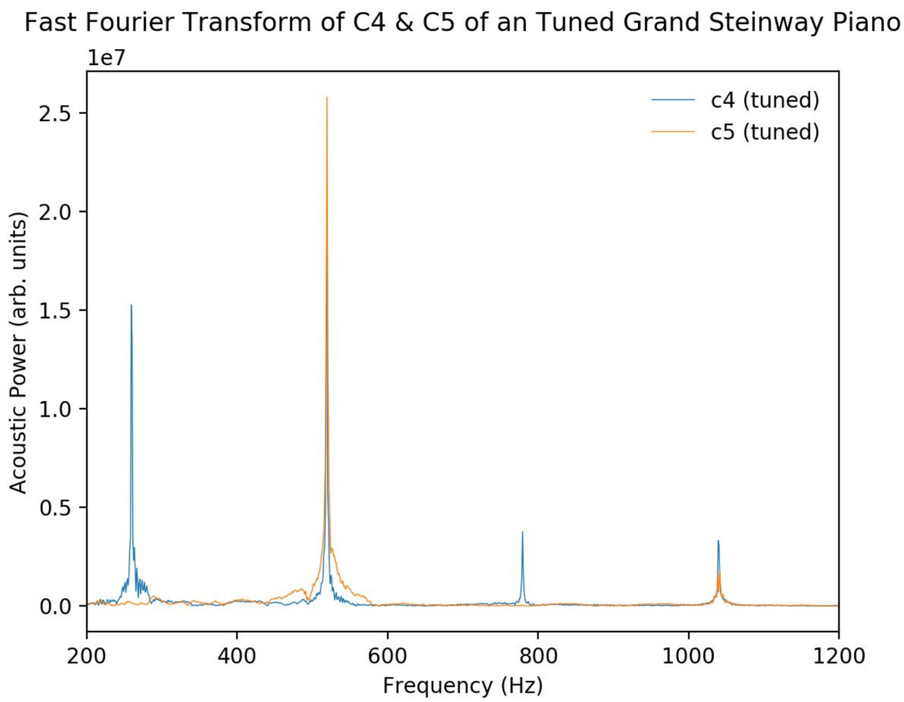

Figure 13: The alignment of the C4 harmonics with C5 fundamentals & harmonics of a tuned Grand Steinway Piano, scaled for comparison. The x-axis is frequency measured in Hertz and the y-axis is the acoustic power.

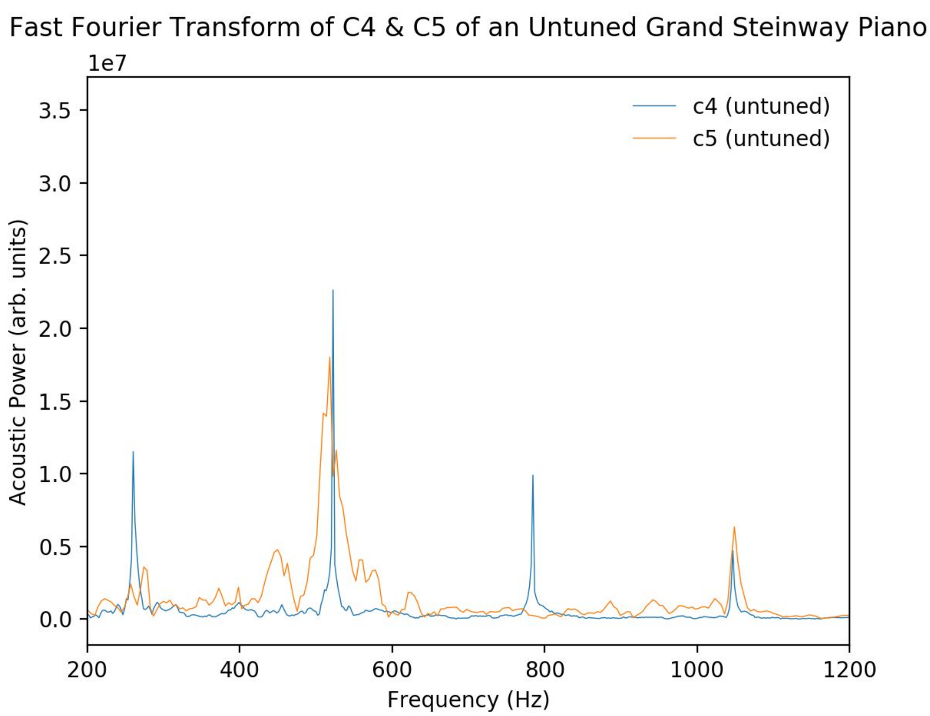

Figure 14: The alignment of the C4 harmonics with C5 fundamentals & harmonics of an untuned Grand Steinway Piano, scaled for comparison. The x-axis is frequency measured in Hertz and the y-axis is the acoustic power.

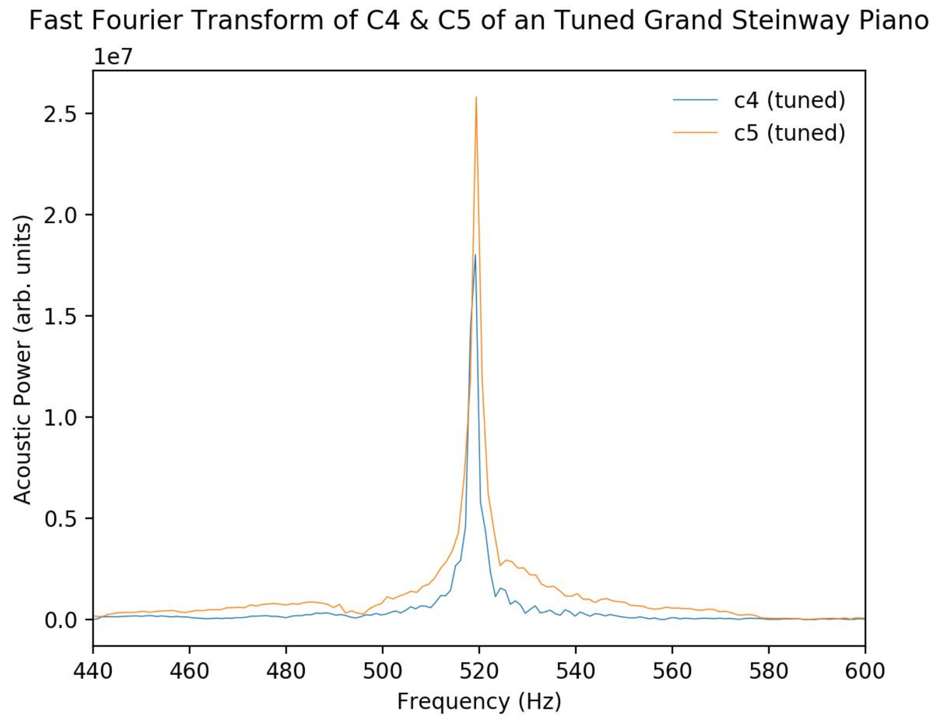

Figure 15: A graph of the tuned Steinway Grand Piano in Figure 13 focused on the 520 Hz pitch. The x-axis is frequency measured in Hertz and the y-axis is the acoustic power.

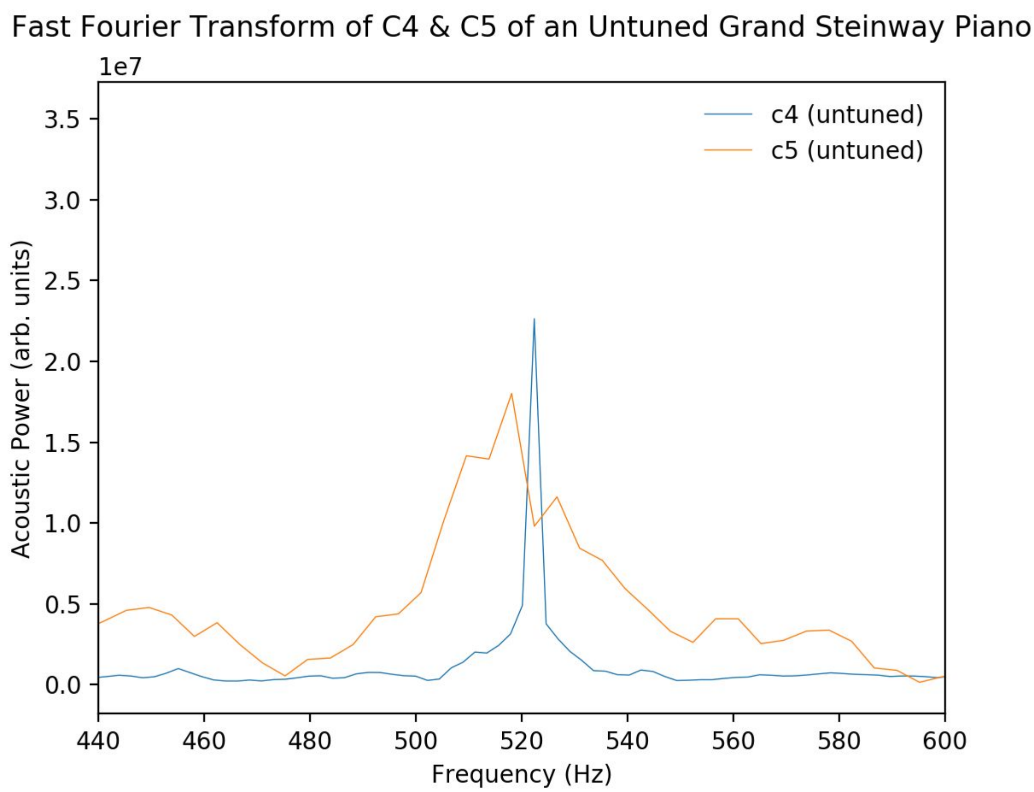

Figure 16: A graph of the untuned Steinway Grand Piano in Figure 14 focused on the 520 Hz pitch. The x-axis is frequency measured in Hertz and the y-axis is the acoustic power.

Overtone Amplitude
The perceived frequency and tone of a note is strongly influenced by the prevalence of its harmonics. When the acoustic power of a note's upper harmonics begin to exceed the acoustic power of its fundamental, the fundamental begins to get overpowered.

The frequency spectra of D2 for a tuned and an untuned Steinway makes an analysis of overtone amplitude easy (Figure 17). For an untuned piano, the upper harmonics of D2 have a much greater acoustic power compared to that of the fundamental frequency, where the third harmonic is the greatest acoustic power. In contrast, the tuned Steinway shows a significantly different trend in amplitudes, where the third harmonic has the lowest acoustic power.

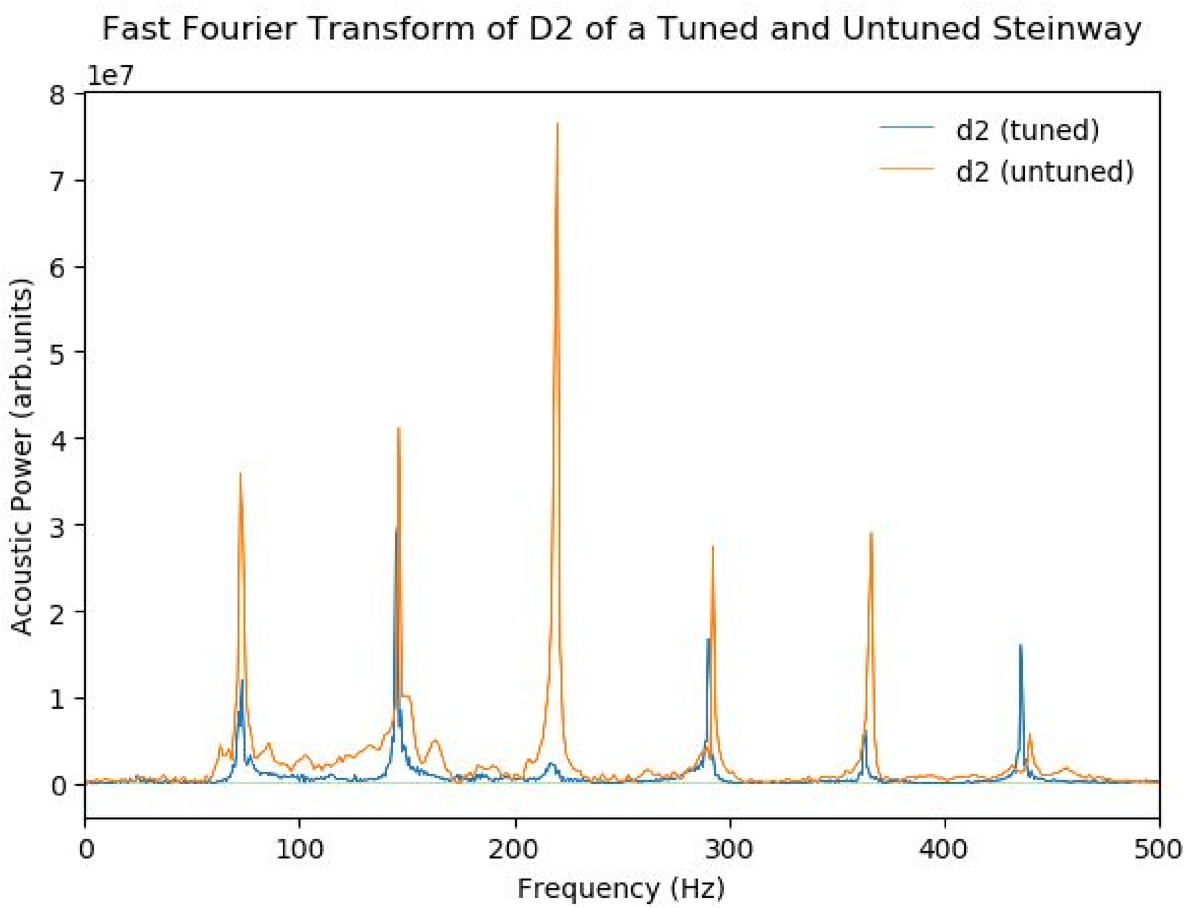

Figure 17: Frequency spectra of D2 for a tuned and an untuned Steinway. For the untuned Steinway, the third harmonic of D2 has an amplitude significantly larger than the fundamental frequency.

Self-Dissonance
An untuned piano can have a pair of peaks, known as a doublet, whereas the tuned piano has a single peak. The doublet shape is caused by dissonance. Doublets cannot form in the lower octaves because each key has one string per note. For notes in the middle and upper octaves, keys

have multiple strings per note. Dissonance occurs when bichord strings are not in tune with each other.[11]

Figures 18 and 19 highlight the frequency peaks for C6 of a tuned and an untuned Yamaha Concert Grand piano. When a piano is out of tune, a listener can often hear beats when the piano is played. This beating is a result of two or more tones of similar frequencies constructively and destructively interfere with each other. In order to eliminate the beating, multiple strings of each piano key should be tuned to the same frequency.[12]

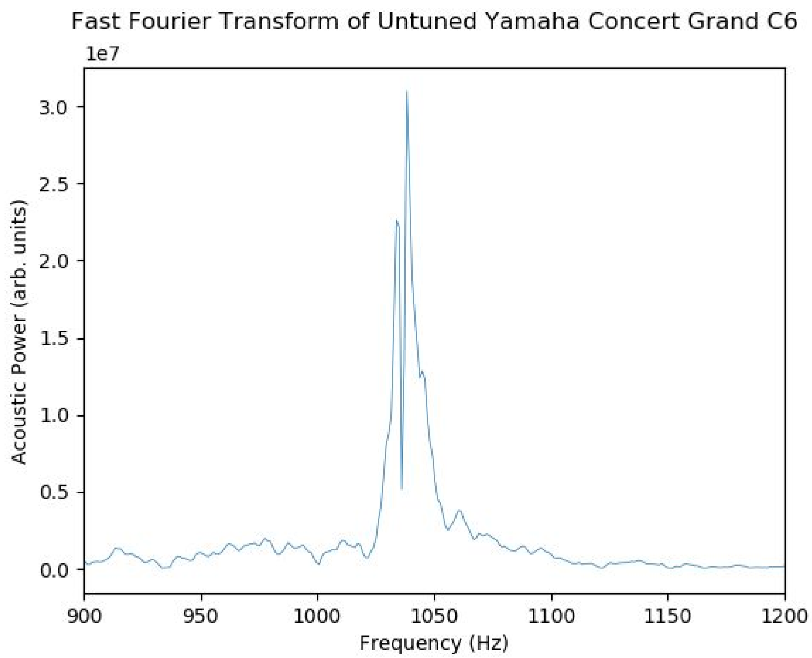

Figure 18: A graph of the fundamental frequency of C6 after FFT of an untuned Yamaha Concert Grand Piano.

[11] Koenig, D. M. (2015). *Spectral analysis of musical sounds with emphasis on the piano*. Oxford: Oxford University Press.
[12] Piano tuning. (2019, February 02). Retrieved May 19, 2019, from https://en.wikipedia.org/wiki/Piano_tuning

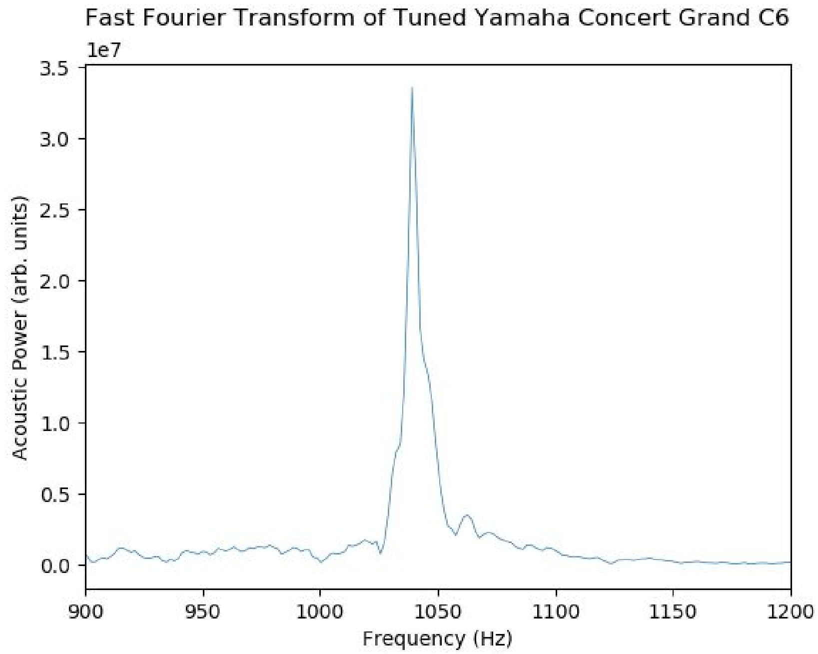

Figure 19: A graph of the fundamental frequency of C6 after FFT of a tuned Yamaha Concert Grand Piano.

Discussion

Throughout the experiments, there could be many possible sources of error. The results may not be completely standardized due to the use of four different devices for data collection. The most promising results are from the Krannert Great Hall Steinway Concert Grand and the Yamaha Concert Grand and were only recorded with one device each. During these tests, the PCB was placed on top of the piano, and as a result might have been subjected to vibrations of the piano structure. It's possible that notes might not have dissipated entirely before recording the next note. To eliminate this source of error in future tests, one device should be used for each recording and should not be placed in direct physical contact with the piano.

Additionally, when analyzing an FFT frequency spectrum, peak frequency values were determined manually, potentially leading to inconsistencies. Moving forward, automating peak analysis on the frequency spectra would help standardize the data analysis process and eliminate this source of error.

Most of data was collected in the Krannert Center for Performing Arts. While the pianos may not be kept in the best conditions, they remain consistently tuned and as a result, there are fewer untuned pianos recorded. The quality of tuning may be another factor that affects results. Tuners use a variety of devices including tuning hammers, tuning mutes, tuning forks, and electronic tuners to reach a desired frequency. This can produce a noticeable difference in the quality of tuning.

To improve future tests, a higher quality microphone can be used. The electret microphone, while inexpensive, does not produce the best results for accurate analysis. Furthermore, recording the barometric pressure, temperature, and humidity for a piano's environment may also prove useful since environmental externalities have a non-negligible effect on the pitch of instruments. Temperature and humidity greatly change not only the quality of tone produced, but also put stress on the instrument's design especially if it is made from wood. The ideal environment for the best pitch from a piano is 22-25℃ and 45-55% humidity. An alternative experiment might focus on how a single piano falls out of tune over time and make use of the devices discussed above.

The results from this analysis can be used for the appraisal of pianos, the training of piano tuners, and verifying tonal quality before concerts. Software could be developed using the methods presented in this paper. This software would take a scale as an input, eliminate background noise, analyze the FFT, generate a Railsback curve, and compare the generated curve to an ideal curve to determine the quality of a piano.

CONCLUSION

The perceived tonal quality of a piano is dependent on more than a frequency shift across each octave. Eight pianos of varying levels of tonal quality were recorded, and the frequency spectra of three octaves were analyzed for each piano. Untuned pianos exhibited large frequency differences, poor octave correspondence, erratic overtone amplitude patterns, and noticeable self-dissonance relative to tuned pianos.

ACKNOWLEDGEMENTS

Special thank you to Professor George Gollin for the equipment, sensors, prototype diagram, and direction and the University Physics Department for funding our course. Thank you to the TAs who helped us along the way in making sure the project was completed in a timely manner. Thanks to all the facilities who provided pianos that were tests and all the technicians and musicians involved.

APPENDIX

Piano	Note	1st (Fundamental) f	2nd f	3rd f	4th f	Strongest Harmonic	1st Actual	1st diff	1st % diff.	2nd diff	2nd % diff.
Concert Yamaha	C4	261.011	521.734	781.115	1042.53	1st	261.63	-0.619	0.236594	-0.288	0.05517
	D4	291.887	583.481	875.379	1169.11	1st	293.66	-1.773	0.603759	-0.293	0.050191
	E4	328.115	654.279	983.204	1311.78	1st	329.63	-1.515	0.459606	-1.951	0.297304
	F4	348.949	695.859	1041.4	1388.31	1st	349.23	-0.281	0.080463	-2.039	0.292163
	G4	389.224	778.108	1169.39	1560.67	2nd	392	-2.776	0.708163	-0.34	0.043677
	A4	439.932	874.61	1313.29	1755.98	1st	440	-0.068	0.015455	-5.254	0.597138
	B4	491.9	982.911	1475.57	1972.09	1st	493.88	-1.98	0.400907	-0.889	0.090364
	C5	523.256	1039.81	1561.55	2083.29	1st	523.3	-0.044	0.008408	-6.702	0.640413

Figure 1: Quantitative results for the fundamental frequencies of Figure 8 are measured in Hz. Each note has a first through fourth measured frequencies and the differences between the actual fundamental frequency. The strongest peak from the FFT is also listed.

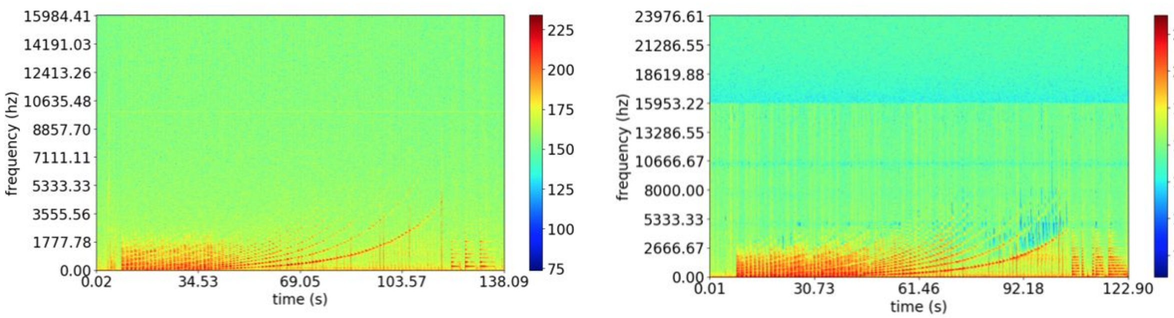

Figure 2: Spectrogram of the 88 piano keys of a tuned piano (left) and untuned (right).

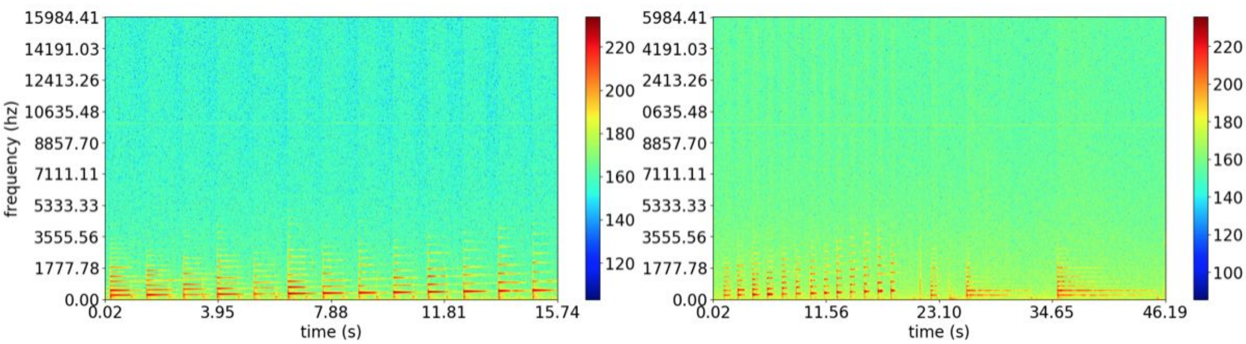

Figure 3: Spectrogram of C4 of a tuned piano (left) and untuned piano